CHAPTER 6 POWERPOINT

AP Calculus

REMEMBER WHEN….

We learned about antiderivatives and slope fields?

DIFFERENTIAL EQUATION

- Equation containing a derivative
- Pointer: Remember to separate the variables and then do the antiderivative
- And another pointer: Don't forget to C

INITIAL VALUE PROBLEM

- This is a problem where you are finding a function when you are given a derivative and an initial value.
- This usually means you are given a point so that you can find the C after you do the antiderivative

WHAT'S THE DIFFERENCE?

- Finding a *particular solution* for a certain initial condition is solving the initial value problem
- Finding EVERY function that satisfies a differential equation is solving the differential equation

SLOPE FIELD

- A slope field is the graph of the curve at a bunch of different points, each with its own initial condition.

INDEFINITE INTEGRAL

- Has no integration limits
- Has +C

CONSTANT MULTIPLE RULE

$$\int kf(x)dx = k\int f(x)dx$$

SUM AND DIFFERENCE RULE

$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$$

- Note: This is not products or quotients!!

▶Page 313 #42

LET ME SHOW YOU

INTEGRATION BY SUBSTITUTION

- You are already experts at this. Remember that substituting may change the integration limits.

SEPARABLE DIFFERENTIAL EQUATIONS

- The key point here is to remember to *separate the variables* before you take the antiderivatives

EXPONENTIAL GROWTH AND DECAY

- Let's think about a quantity, y, that increases or decreases at a rate proportional to the amount present:

$$\frac{dy}{dx} = ky$$

INITIAL CONDITION: $Y=Y_0$ WHEN $T=0$

$$\frac{dy}{dt} = ky$$

$$\int \frac{dy}{y} = \int k\,dt$$

$$\ln|y| = kt + C$$

$$e^{\ln|y|} = e^{kt+C}$$

$$|y| = e^{c}e^{kt}$$

$$y = \pm e^{c}e^{kt}$$

$$y = Ae^{kt}$$

Let $A=\pm e^{C}$

- Does this formula look familiar?

$$y = Ae^{kt}$$

LAW OF EXPONENTIAL CHANGE

- If y changes at a rate proportional to the amount present and $y=y_0$ when $t=0$ then
- $y=y_0\, e^{kt}$
- where $k>0$ represents growth and $k<0$ represents decay. $k=$ constant rate.

COMPOUND INTEREST

- For discrete time periods

$$A(t) = A_0\left(1 + \frac{r}{k}\right)^{kt}$$

- For continuous compounding

$$A(t) = A_0 e^{rt}$$

RADIOACTIVITY

- Radioactive decay if k>0 is

$$y = y_0 e^{-kt}$$

- Note:Carbon-14 dating uses 5700 year half-life

RADIOACTIVITY

Half-life is time to decay to half the original amount and is equal to

$$\frac{\ln 2}{k}$$

NEWTON’S LAW OF COOLING

- The rate of temperature change is proportional to the difference between its temperature and that of its surroundings. Applications? Cooling off a can of pop, for example.

NEWTON'S LAW OF COOLING

- If T=temperature at time t
- And T_s=surrounding temperature
- And T_0=temperature at time t=0

$$T - T_s = (T_0 - T_s)e^{-kt}$$

RESISTANCE PROPORTIONAL TO VELOCITY

The assumption that resistance (like air resistance) is proportional to velocity can be written

$$F = ma = m\frac{dv}{dt}$$

Solution:

$$m\frac{dv}{dt} = -kv$$

$$\frac{dv}{dt} = -\frac{k}{m}v, k > 0$$

$$v = v_0 e^{-(\frac{k}{m})t}$$

- Let’s read together from page 342 - 346

EXPONENTIAL POPULATION GROWTH MODEL

Homework

- 313(25, 27, 30, 33, 38, 39, 42, 45, 51, 52, 61)
- 314 (49, 53-55, 57, 62)
- 322(24-42 mult 3, 13, 23, 25)
- Finish up yesterday’s work and do 323 (43, 44)
- 338(1-9odd,12,14)
- 338(15-33, mult 3)
- 358 (33, 34, 54) + 347(1-13,odd)