

# CHAPTER 6 POWERPOINT

AP Calculus

A series of several parallel white lines of varying thicknesses, slanted diagonally from the bottom-left towards the top-right, located in the lower right quadrant of the slide.

# REMEMBER WHEN....

We learned about  
antiderivatives and slope  
fields?



# DIFFERENTIAL EQUATION

- ▶ Equation containing a derivative
  - ▶ Pointer: Remember to separate the variables and then do the antiderivative
  - ▶ And another pointer: Don't forget to C
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# INITIAL VALUE PROBLEM

- ▶ This is a problem where you are finding a function when you are given a derivative and an initial value.
  - ▶ This usually means you are given a point so that you can find the  $C$  after you do the antiderivative
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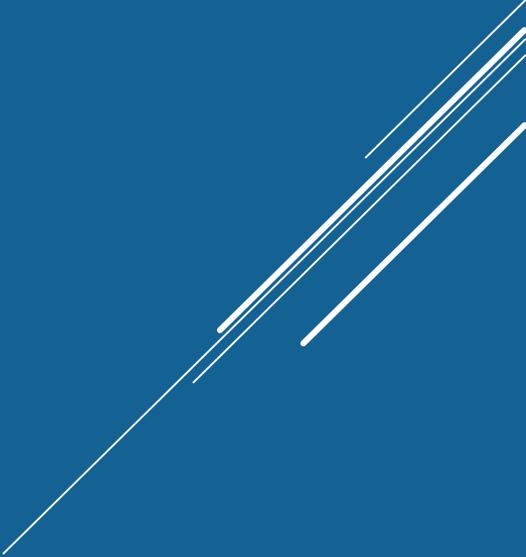
# WHAT'S THE DIFFERENCE?

- ▶ Finding a *particular solution* for a certain initial condition is solving the initial value problem
  - ▶ Finding EVERY function that satisfies a differential equation is solving the differential equation
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## SLOPE FIELD

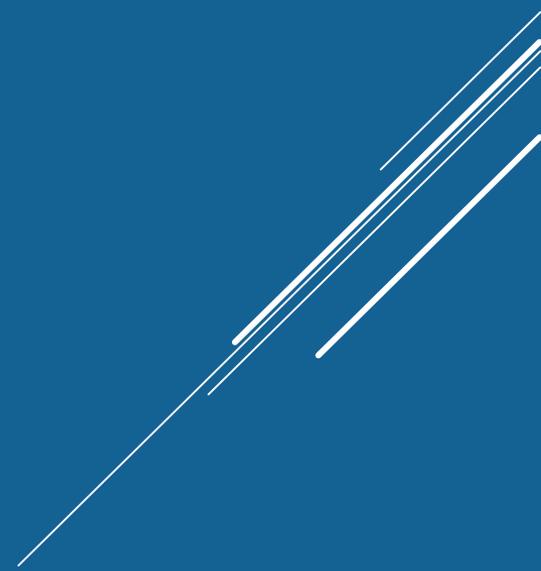
- ▶ A slope field is the graph of the curve at a bunch of different points, each with its own initial condition.
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- A decorative graphic consisting of several parallel white lines of varying thicknesses, slanted diagonally from the bottom right towards the top right, located in the lower right quadrant of the slide.

# INDEFINITE INTEGRAL

- ▶ Has no integration limits
  - ▶ Has  $+C$
- 

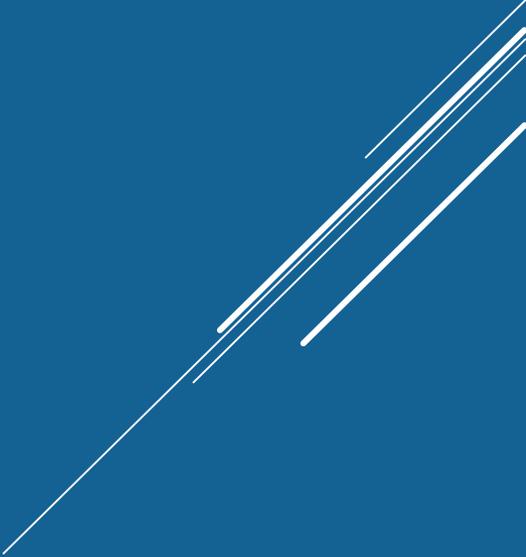
## CONSTANT MULTIPLE RULE

$$\int kf(x)dx = k \int f(x)dx$$



# SUM AND DIFFERENCE RULE

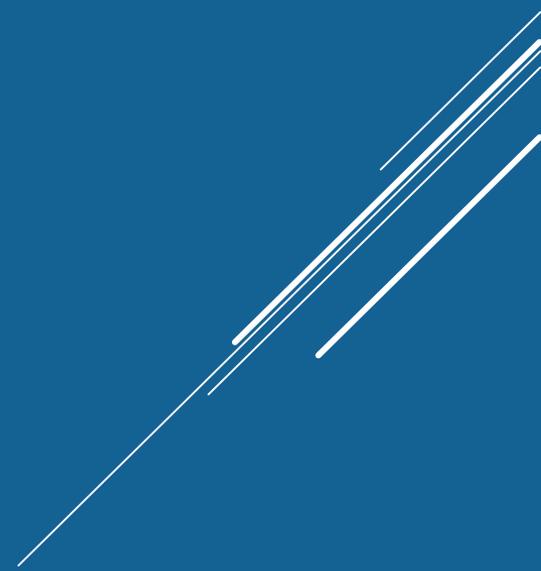
$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$$

- ▶ Note: This is not products or quotients!!
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▶ Page 313

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LET ME SHOW YOU



## INTEGRATION BY SUBSTITUTION

- ▶ You are already experts at this. Remember that substituting may change the integration limits.
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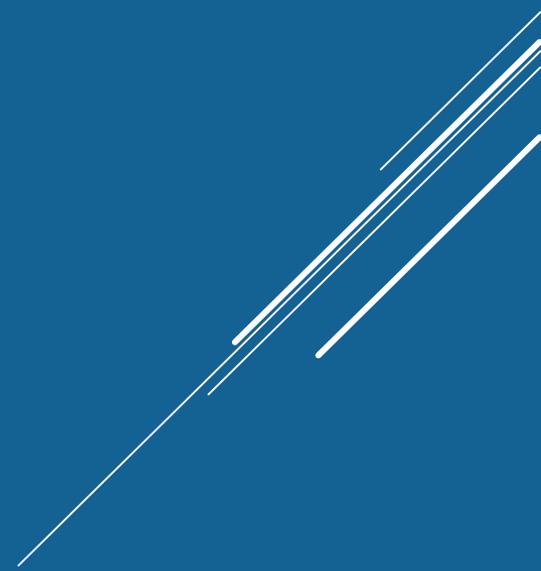
## SEPARABLE DIFFERENTIAL EQUATIONS

- ▶ The key point here is to remember to *separate the variables* before you take the antiderivatives
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# EXPONENTIAL GROWTH AND DECAY

- ▶ Let's think about a quantity,  $y$ , that increases or decreases at a rate proportional to the amount present:

$$\frac{dy}{dx} = ky$$



INITIAL CONDITION:  $Y=Y_0$  WHEN  $T=0$

$$\frac{dy}{dt} = ky$$

$$\int \frac{dy}{y} = \int k dt$$

$$\ln|y| = kt + C$$

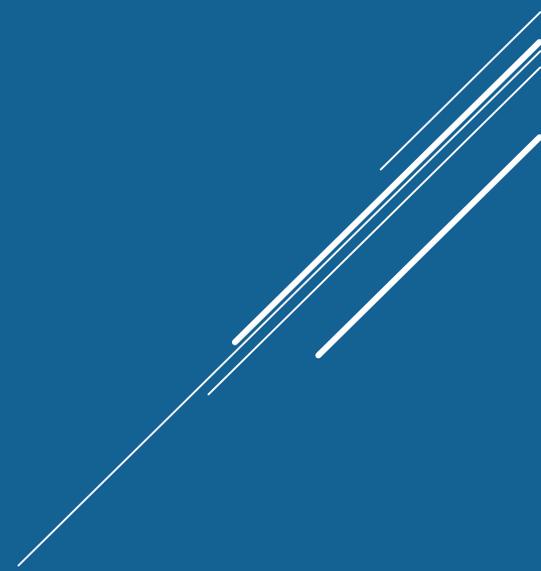
$$e^{\ln|y|} = e^{kt+C}$$

$$|y| = e^c e^{kt}$$

$$y = \pm e^c e^{kt}$$

$$y = Ae^{kt}$$

Let  $A = \pm e^C$



▶ Does this  
formula look  
familiar?

$$y = Ae^{kt}$$


## LAW OF EXPONENTIAL CHANGE

- ▶ If  $y$  changes at a rate proportional to the amount present and  $y=y_0$  when  $t=0$  then
- ▶  $y=y_0 e^{kt}$
- ▶ where  $k>0$  represents growth and  $k<0$  represents decay.  
 $k$ = constant rate.

# COMPOUND INTEREST

- ▶ For discrete time periods

$$A(t) = A_0 \left(1 + \frac{r}{k}\right)^{kt}$$

- ▶ For continuous compounding

$$A(t) = A_0 e^{rt}$$

# RADIOACTIVITY

- ▶ Radioactive decay if  $k > 0$  is

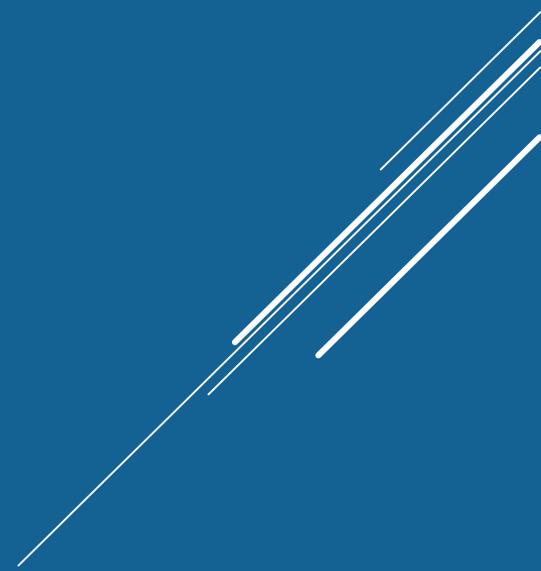
$$y = y_0 e^{-kt}$$

- ▶ Note: Carbon-14 dating uses 5700 year half-life

# RADIOACTIVITY

Half-life is time to decay to half the original amount and is equal to

$$\frac{\ln 2}{k}$$

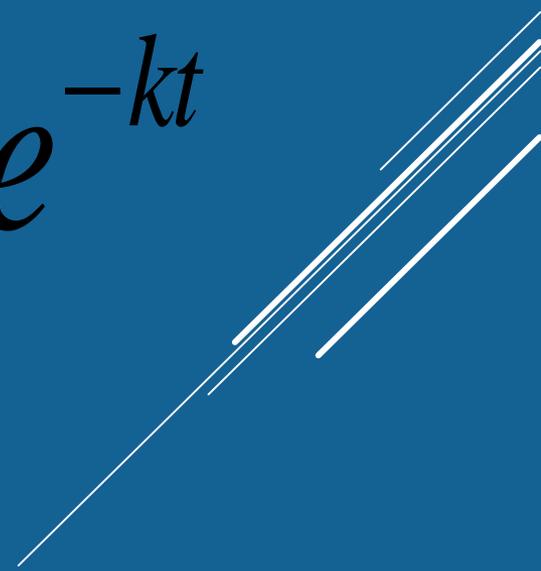


# NEWTON'S LAW OF COOLING

- ▶ The rate of temperature change is proportional to the difference between its temperature and that of its surroundings. Applications? Cooling off a can of pop, for example.
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# NEWTON'S LAW OF COOLING

- ▶ If  $T$ =temperature at time  $t$
- ▶ And  $T_s$ =surrounding temperature
- ▶ And  $T_0$ =temperature at time  $t=0$

$$T - T_s = (T_0 - T_s)e^{-kt}$$


# RESISTANCE PROPORTIONAL TO VELOCITY

The assumption that resistance (like air resistance) is proportional to velocity can be written

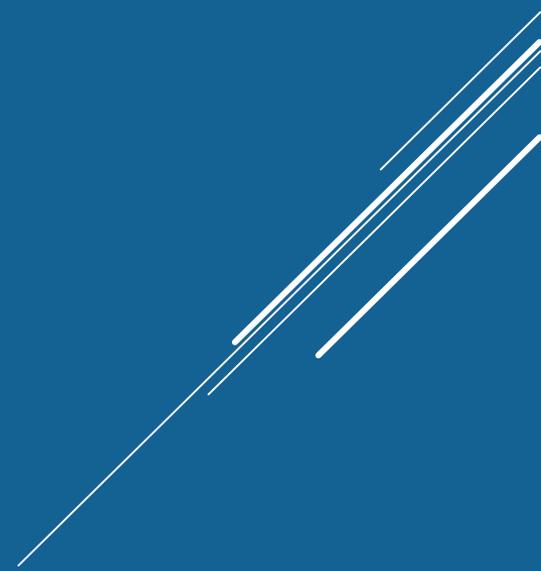
$$F = ma = m \frac{dv}{dt}$$

Solution:

$$m \frac{dv}{dt} = -kv$$

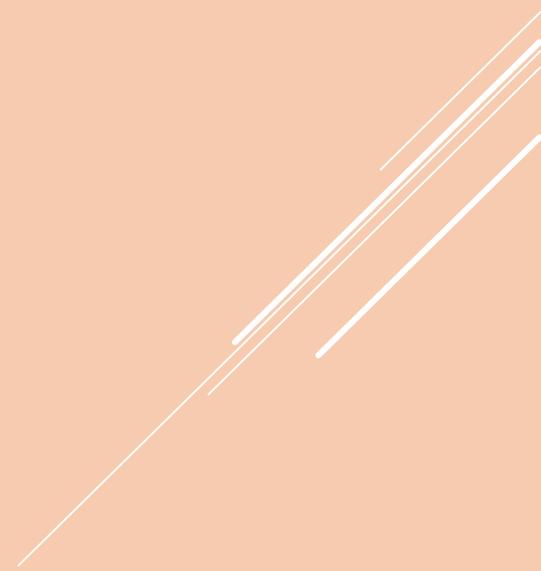
$$\frac{dv}{dt} = -\frac{k}{m} v, k > 0$$

$$v = v_0 e^{-\left(\frac{k}{m}\right)t}$$



▶ Let's read  
together from  
page 342 -  
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EXPONENTIAL POPULATION  
GROWTH MODEL



# Homework

- 313(25, 27, 30, 33, 38, 39, 42, 45, 51, 52, 61)
  - 314 (49, 53-55, 57, 62)
  - 322(24-42 mult 3, 13, 23, 25)
  - Finish up yesterday's work and do 323 (43, 44)
  - 338(1-9 odd, 12, 14)
  - 338(15-33, mult 3)
  - 358 (33, 34, 54) + 347(1-13, odd)
- 